

1 Simulation-based high resolution fire danger
2 mapping using deep learning

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13 **Abstract:** Wildfire occurrence and behavior are difficult to predict very
14 locally for the next day. In the present work, we use an artificial neural net-
15 work emulator called DeepFire, trained on the basis of simulated fire sizes,
16 and study its application to fire danger mapping using actual weather fore-

casts. Experimental analysis is based on DeepFire forecasts for 13 relatively big fires that occurred in Corsica and corresponding forecasts based on a fire danger index used in operational conditions. A comparative analysis of both indices is presented, highlighting the differences in terms of precision and expected results of such predictions. Forcing weather forecasts used as input have high spatial resolution and high frequency, which also applies to the fire danger predictions. Additionally, input uncertainty is propagated through DeepFire, resulting in ensembles of emulated fire size. Eventually, several approaches are proposed to analyze the results and help in investing assessment of next-day fire danger using this new simulation-based prediction system.

Keywords: wildfire simulation, deep learning, fire danger, potential fire size, fire weather, probability distributions

1 Introduction

Wildfire occurrence and behavior is difficult to predict accurately. The environmental conditions such as weather and the type(s) of vegetation involved may be known to some extent, but the time, location, and cause of fire occurrence are rarely known ahead of time. A common approach in wildfire prediction consists in assessing fire danger, a general term that expresses both fixed and variable factors of the fire environment that influence the ease of ignition, rate of spread and difficulty of control.

Fire danger rating systems include assessment of one or several fire danger indices, which are used among other information to provide a rating, i.e. a class whose possible values may include “low”, “moderate”, “severe”, “extreme”, etc., but the notion of “rating” may also refer to the scalar indices composing the system. Regardless of the type of quantity used to assess it, fire danger relates to the proneness for ignition, spread and/or intensity of a wildfire according to the state of the vegetation and its environment at a given time, therefore reflecting how difficult it may be to control fire. Development of fire danger rating methods has led to the implementation of systems at the national scale about 50 years ago in Canada and in the US, with the Canadian Forest Fire Danger Rating System (CFFDRS [Lawson and Armitage \(2008\)](#), whose development started in 1968) and the National Fire Danger Rating System (NFDRS [Bradshaw et al. \(1984\)](#), first published in 1972, but whose development as well as fire danger rating methods dates

52 further back, see for instance [Hardy and Hardy \(2007\)](#)).

53 Maps of fire danger ratings are usually generated every day to assess
54 the situation of the current day and forecast the situation for the day(s)
55 to come. Such maps may be available among other data via internet-based
56 information systems; for instance, covering the US as part of the Wildland
57 Fire Assessment System (WFAS [Burgan et al. \(1997\)](#))¹, covering Canada as
58 part of the Canadian Wildland Fire Information System (CFWIS)², covering
59 Europe and the Mediterranean area as part of the European Forest Fire
60 Information System (EFFIS)³, or even covering the globe as part of the
61 Global Wildfire Information System (GWIS)⁴, which builds on activities of
62 EFFIS.

63 Calculation of the aforementioned fire danger indices mostly depends on
64 weather inputs and, to some extent, fuel moisture, which is generally derived
65 from weather information. In the CFFDRS, the final output is a composite
66 Fire Weather Index (FWI), that depends on such inputs, more precisely wind
67 speed, air temperature, relative humidity and rainfall over the previous 24
68 hours, as well as the values of the indices from the day before. Although one
69 could consider computing the indices at high temporal resolution, they are
70 generally computed to represent peak, or at least high, fire weather conditions
71 in a given day. Also, although the FWI was calibrated to describe fire behav-

¹<https://www.wfas.net/>, last checked 2021.02.01

²<https://cwfis.cfs.nrcan.gc.ca/maps/fw>, last checked 2021.02.01

³<https://effis.jrc.ec.europa.eu/>, last checked 2021.02.01

⁴<https://gwis.jrc.ec.europa.eu/>, last checked 2021.02.01

ior in a jack pine stands, it has been implemented in many other countries despite the differences in vegetation and climate where it showed reasonable performance (see for instance [Giuseppe et al. \(2020\)](#) and references therein). Forecasted maps of such indices generally have a temporal resolution of one day even if the underlying weather forecasts have higher temporal resolution, but the temporal extent and spatial resolution are generally the same.

Weather forecasts covering a smaller area may have higher spatial resolution: for instance the regional AROME limited area model in use at Météo-France has a spatial resolution of 1.3 km, covers France and part of neighboring countries, and allows for the computation of fire danger index at much higher resolution. AROME is also an operational system used as a reference in this study. Meteorological assistance for forest fires started in the 60s and the Météo-France methodology has been gradually evolving over the years, depending on the research but also on the feedback of the various departments of Civil Protection and the ONF who fight against forest fires.

In summer, the Mediterranean vegetation is evolving between a growth phase in spring to a dormancy phase between mid-summer and early fall. When the scrubland's shrubs are falling over to this vegetative state, there is a very small quantity of water in leaves and thus the Mediterranean vegetation is becoming extremely flammable by dry weather. The expertise of weather fire danger requires, firstly, the monitoring of drought to evaluate whether or not the vegetation is prone to big summer fires.

In this configuration, at Météo-France, the forecaster assesses weather fire

95 danger with 3 main indices; FWI: for vegetation in low or moderate drought,
96 IPse: a local rate of spread index for very dry vegetation and IEP (Indicateur
97 d'éclosion et Propagation, French for "Ignition and spread indicator"): for
98 dead or dormancy vegetation for which the water content is directly linked
99 with air moisture and therefore Fine Fuel Moisture Content (FFMC). All
100 those indices are calculated at the maximum of the day based on hourly val-
101 ues (in which case, they are post-fixed with an x, leading to FWIx, IPsex,
102 and IEPx). Weather fire danger depends on wind speed and highest temper-
103 atures/lowest air moisture, which can happen at different moments during
104 the day depending on the local meteorological context; for instance, strong
105 and dry winds can happen at any moment in the day and even in the night
106 in Mediterranean areas.

107 Instead of relying on empirically calibrated formulas to compute indices
108 that are, for the most part, unitless, a promising strategy is to rely on a
109 large number of simulations of fire spread using weather forecasts as input,
110 therefore providing a more "concrete" representation of fire spread via sim-
111 ulated burned surfaces. Two interesting methods following such a strategy,
112 although applied at different time scales than day to day predictions, are
113 burn probability (BP) modeling and ensembles of simulations for specific fire
114 cases.

115 BP modeling consists in propagating input distributions of environmen-
116 tal characteristics (weather and fuels) and fire characteristics (ignition, fire
117 duration, number of fires in a year) via several simulations of wildland fire

118 ignition and spread, this process being repeated independently several times
119 via a MC method to represent potential scenarios of wildland fire activity in
120 a region and/or a country. In this context, the output BP maps represent the
121 probability of local areas to be burned by a fire (typically over the course of a
122 year). Notable implementations include Burn-P3 [Parisien et al. \(2005\)](#) which
123 relies on the Prometheus solver [Tymstra et al. \(2010\)](#) and FSim [Finney et al.](#)
124 [\(2011b\)](#) which relies on FARSITE [Finney \(1998\)](#). A large number of studies
125 have proposed and/or applied BP modeling (see for instance [Parisien et al.](#)
126 [\(2019\)](#) and references therein).

127 Similarly, ensembles of simulations for specific fire cases allow to assess
128 “burn probabilities”, following a Monte Carlo approach that accounts for
129 input uncertainty in fire spread simulations, notably regarding weather vari-
130 ables. A notable implementation of this method is the FSPro system [Finney](#)
131 [et al. \(2011a\)](#), which relies on an ensemble of FARSITE simulations. Due to
132 knowledge of the ignition location, the burn probability maps resulting from
133 such ensembles cover a much smaller area than maps of BP modeling.

134 From an operational perspective, BP modeling can be seen as a decision
135 support tool for “long-term” planning, whereas ensembles of simulations for
136 specific fire cases apply to “crisis” situations. In “short-term” predictions,
137 however, the input distributions should be more representative of the ex-
138 pected weather conditions as in ensembles of simulations for specific fire
139 cases, but should also account for the lack of knowledge regarding future
140 ignitions over a vast territory, as in BP modeling. Although BP modeling

141 methods could be adapted to account for daily weather forecasts, their com-
142 putational requirement due to the large number of simulations involved is
143 too high for use in an operational context.

144 The approach developed in this work proposes to use wildland fire sim-
145 ulations as basis for short-term assessment of fire danger, while relying on
146 deep learning to reduce the computational cost of running a high number
147 of simulations. A deep neural network (DNN) referred to as *DeepFire* in
148 the following is used to approximate the potential fire size returned by a fire
149 spread simulator.

150 DeepFire is able to compute a high number of fire size estimations in
151 a short amount of time, and could generate fire danger maps at both high
152 spatial resolution (approximately 80 m) and high frequency (10-minute time
153 step) in a operational context. The fire size computations could be obtained
154 using a fire spread simulator, but would require several days in this context.
155 Compared to traditional fire danger indices, two major differences of Deep-
156 Fire are that it accounts not only for weather but also for the influence of
157 terrain on fire spread, and that fire danger is expressed as a physical quan-
158 tity, namely a surface area (in hectares). Its output and overall design are
159 therefore quite different from the CFFDRS, for instance. It is also possible
160 to run an ensemble of DeepFire predictions to quantify uncertainty, although
161 at lower spatial resolution (640 m). Compared to BP modeling, a noteworthy
162 difference is that DeepFire focuses on “sources” of fire spread (i.e. how large
163 will be the burned surface, assuming that an ignition will occur at a specific

location), while burn probabilities represent “sinks” of fire spread (i.e. how often a specific location is likely to be burned, as a result of several scenarios of fire spread each with variable ignition location).

A key aspect of this work is the the availability of high-resolution weather forecasts, which is arguably a major limitation for application of a BP approach to short-term predictions. In the frame of this study, a limited-area model running at 600-m resolution is used to generate a forecast of surface wind, fuel moisture, and air temperature with a time extent of 42 hours. In order to be representative of operational applications, this high-resolution weather forecast was run daily (at midnight) from 10/05/2017 and was available around 11:00. To account for rapidly changing conditions, a high-frequency simulation output with a time step of 10 minutes is provided, resulting in time-series of 253 spatial fields of weather variables, which are forecasted every day.

Section 2 details the method that is used to obtain predictions of fire danger with DeepFire and with IEP. Analysis of forecasts for days were relatively large fires occurred is presented in Section 3. A detailed analysis is first provided for one fire, then more synthetically for 12 other fires. Section 4 addresses the question of generating daily maps that indicate the danger for the coming day in order to summarize the high number of maps resulting from the high forecast frequency. Finally, the main conclusions of this study and some perspectives are given in Section 5.

186 2 Method

187 2.1 DeepFire: fire size emulation using a deep neural 188 network

189 The new approach proposed to quantify fire danger consists in predicting,
190 for a given time and location, the size of the burned surface that would
191 result from one hour of free wildfire spread after an early stage where the
192 fire has already ignited and spread over about one acre. Ignition probability
193 is assumed homogeneous, except for locations without vegetation where it
194 is considered null. A duration of one hour is generally more than the time
195 necessary for the first attack on the fire to be carried out, even more so if one
196 assumes that the fire has been detected in the early stage. These simplifying
197 assumptions imply that fire danger mostly expresses potential for fire spread
198 if it is not attacked rather than potential for ignition.

199 In the present study, the one-hour fire size is estimated using the DNN
200 *DeepFire*. Its architecture is “hybrid” in the sense that it has two types of
201 input data. The first type is similar to an image: it is composed of four
202 rasters describing the landscape with a spatial resolution of about 80 m in a
203 square of 20 km \times 20 km centered around the ignition location. One raster
204 represents elevation and the other three are derived from the same land cover
205 data. Aside from non-burnable areas, each type of land use is matched with
206 a fuel behavior model, for a total of 13 different fuel models. The three fields
207 other than elevation therefore represent the variable parameters of the fuel

Input	Symbol	Unit	Type	Range	Constraint
Wind speed	(W_x, W_y)	m s^{-1}	Raw	$[-35, 35]^2$	Euclidean norm ≤ 35
Fuel moisture content (dead fuel)	m_c		Raw	$[0.04, 0.3]$	
Heat of combustion perturbation	ΔH	MJ kg^{-1}	Additive	$[-5, 5]$	
Particle density perturbation	ρ_p	kg m^{-3}	Additive	$[-300, 300]$	
Fuel height perturbations	h	m	Multiplicative	$[0.4, 1.6]^{13}$	
Fuel load perturbations	σ_f	kg m^{-2}	Multiplicative	$[0.4, 1.6]^{13}$	
Surface-volume ratio perturbations	S_v	m^{-1}	Multiplicative	$[0.4, 1.6]^{13}$	
Relative ignition point coordinates	$(\delta x, \delta y)$	m	Raw	In a fuel cell	

Table 1: Scalar inputs of DeepFire. In the case of perturbations, the symbol corresponds to the perturbed quantity, and the perturbation of this quantity can be either additive or multiplicative. The range indicates the boundaries of the domain of definition with two components for the wind and 13 components in the last three rows (one row per fuel type).

models, namely, the height h , the load σ_f , and the surface-volume ratio S_v . The second type of inputs are scalars, listed in Table 1, that describe physical quantities influencing fire behavior: the wind speed vector (W_x, W_y) , the FMC m_c , and coefficients used to perturb the default values of fuel model parameters. Two fuel parameters, namely the heat of combustion ΔH and the particle density ρ_p , have the same default value for all fuel models and only have one additive perturbation coefficient each. The other parameters (h , σ_f , and S_v) depend on the fuel model and each have one multiplicative perturbation coefficient per fuel type, which, including the previous two coefficients, amounts to 41 perturbation coefficients. The remaining scalar inputs are two coordinates that locate the ignition point inside the fuel cell of the land cover raster where fire starts, but given the small size of the fuel cells, these two inputs barely have any influence on fire size estimation.

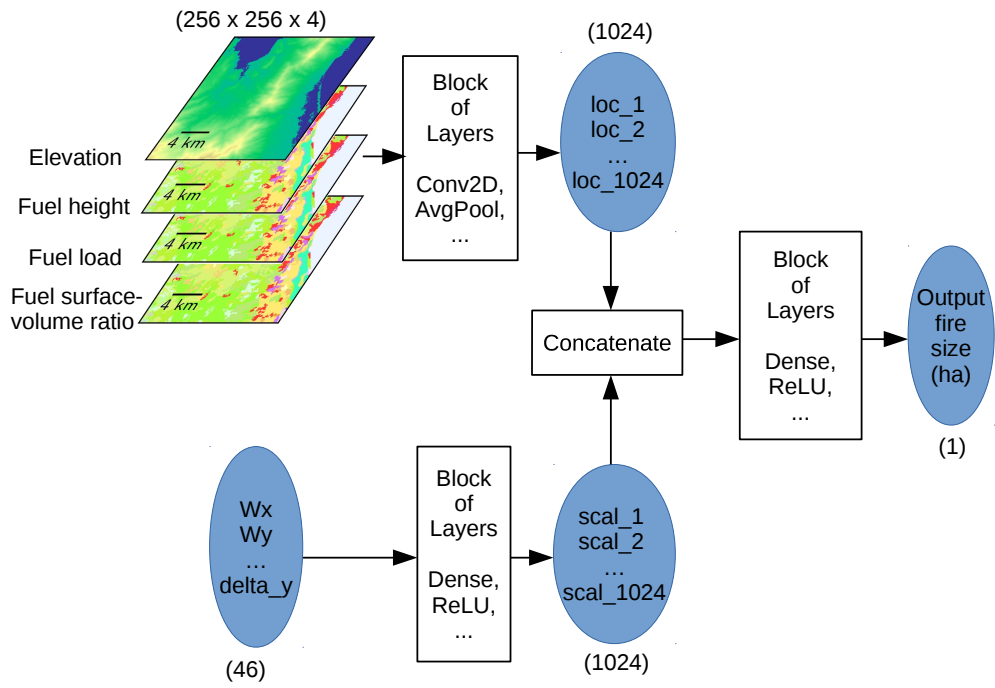


Figure 1: Architecture of the DeepFire neural network.

Each type of input is first processed differently by DeepFire, a summarized
 representation of the neural network architecture being shown in Figure 1.
 Several convolutional layers and average pooling layers (among others) are
 applied to the rasters while the scalars mostly go through a dense layer and
 a rectified linear unit (ReLU) layer. This results in two intermediate vectors
 of size 1024: the first vector only depends on the terrain around ignition
 point, whereas the second vector depends on the scalar inputs. The rest
 of the network consists in concatenating both vectors, and applying several
 dense and ReLU layers, for finally providing an estimate of the fire size.
 The parameters of the network were fitted so that DeepFire returns a good
 approximation of the one-hour fire size that would be simulated by the fire
 spread solver ForeFire [Filippi et al. \(2010\)](#) using the rate of spread model
 of Rothermel [Rothermel \(1972\)](#) with the same inputs. In the simulations,
 the initial burned surface is a 0.45-ha octagon, which is also the minimum
 returned by both ForeFire and DeepFire (this generally happens when the
 FMC is close to the moisture of extinction). DeepFire is an emulator (aka,
 metamodel or surrogate model) of the one-hour fire size returned by ForeFire,
 that is to say a model that computes this output considerably faster. The
 first intermediate vector of “location” is pre-computed for each fuel cell in
 the region studied (about 1.2 million fuel cells for Corsica island). Based
 on these pre-computed vectors, the one-hour fire size can be computed for
 all cells in about one minute for any ignition location, which is thousands
 of times faster than with ForeFire and makes daily fire danger mapping

244 in operational conditions technically possible. More details on the neural
245 network and its use for emulation of fire spread simulation can be found
246 in [Allaire et al. \(2021a\)](#).

247 **2.2 Deterministic predictions of fire danger maps**

248 The design of DeepFire allows for fire danger mapping of Corsica island at
249 high spatial resolution. First, for a given fuel cell, the intermediate location
250 vector (that was pre-computed) is used as input of the concatenate block.
251 The ignition point is assumed to be located at the center of the fuel cell, which
252 defines δ_x and δ_y . Then, scalar fuel parameters perturbations are set to 0 and
253 1 for additive and multiplicative coefficients, respectively, which corresponds
254 to no alteration of the default values of the fuel model parameters. Finally,
255 wind speed and FMC are quantities that vary during the day, so they are
256 estimated based on a weather forecast. A spatial interpolation can be carried
257 out to determine the value of these three inputs at the center of a given fuel
258 cell to address differences in spatial resolution and/or coordinate system (in
259 the present study, a barycentric interpolation is used). In extreme cases
260 where the wind speed norm is higher than 35 m s^{-1} , wind speed is reduced to
261 this threshold because DeepFire was not trained based on simulations outside
262 this scope. Similarly, when the FMC is lower than 0.04, this lower bound
263 is used as input of DeepFire. Also, the minimum output value (0.45 ha)
264 is automatically returned when the FMC is higher than 0.3, which is the
265 moisture of extinction used in ForeFire simulations. The computation of the

266 DeepFire output with the aforementioned inputs corresponding to each fuel
 267 cell of the map and each forecast time of the weather forecast results in a
 268 time sequence of fire danger maps at high spatial resolution. Given that
 269 no perturbations are applied to the scalar inputs, this is referred to as a
 270 *deterministic prediction* of fire danger in the following.

Wind speed \ FFMC	≤ 80]80, 85]]85, 89]]89, 93]]93, 96]	> 96
≤ 10 kt	low (1)	low (1)	minor (2)	moderate (3)	severe (4)	severe (4)
]10 kt, 20 kt]	low (1)	minor (2)	moderate (3)	severe (4)	severe (4)	very severe (5)
]20 kt, 30 kt]	low (1)	minor (2)	moderate (3)	severe (4)	very severe (5)	very severe (5)
> 30 kt	minor (2)	moderate (3)	severe (4)	very severe (5)	very severe (5)	very severe (5)

Table 2: Computation of the IEP based on the norm of wind speed (in knots) and the FFMC.

271 Using the same weather forecast as for DeepFire, a time sequence of fire
 272 danger maps at high spatial resolution based on the IEP can be obtained
 273 similarly. IEP is one of the operational danger rating indicator in France
 274 that serves for both summer and winter. The design of IEP is mostly em-
 275 pirical and based on phenological assumptions. In the autumn, as soon as
 276 days become shorter and temperatures lower, the vegetation transforms it-
 277 self again. Deciduous trees lose their leaves which accumulate on soil and the
 278 forest litter thicken. Other trees, shrubs and bushes ‘hibernate’ and become
 279 in dormancy with a very low quantity of sap/water in their leaves. At the
 280 same time, herbaceous are at the end of their cycle and die. In addition
 281 to this traditional and seasonal dormancy, with climate warming, more and
 282 more vegetation dies with huge and record summer drought. The dead or

283 dormancy vegetation water content is directly linked with air humidity (as
 284 a sponge effect). In this case, the IEPx (daily maximum) is the most rele-
 285 vant indicator. In the 2010s, some huge fires occurred with moderate or just
 286 severe weather fire conditions in places where dead vegetation were numer-
 287 ous. By studying indices and conditions in a file constituted by more than
 288 5000 summer alerts from 2001, Météo-France set up an indicator by crossing
 289 FFMC, correlated with outbreak potential, and wind. Since then, by em-
 290 piricism and following winter and summer situations, with Fire Brigades and
 291 National Forest officers, it was concluded that this indicator was the most
 292 relevant indicator for winter fires and a good complement in summer for dead
 293 or dormancy vegetation in very dry conditions (as it was the case in summer
 294 2017 in Corsica).

295 The IEP has five possible categories ranging from “low” to “very severe”
 296 that only depend on the norm of wind speed and the FFMC as determined
 297 from Table 2. Let us underline that both FMC and FFMC are diagnostic
 298 variables of the weather forecast, i.e. they are deduced from the other vari-
 299 ables. The FFMC is generally based on the forecast for 12:00 in local time,
 300 but this method does not account for variations of fuel moisture during the
 301 day, whereas the forecasted FMC used as input of DeepFire may vary over
 302 time. For consistency, we decided not to compute the input FFMC of the
 303 IEP in the “standard” way but based on the value of FMC m_c , according to

304 Equation (2.1):

$$\text{FFMC} = 59.5 \times \frac{250 - m_c}{147.2 + m_c}, \quad (2.1)$$

305 which is the final equation intervening in calculation of the FFMC in the
 306 Canadian system (cf. Equation (10) in Van Wagner and Pickett (1985),
 307 where the other equations are used to provide an estimate of m_c).

Fire size (ha)	< 0.1	[0.1, 4.0[[4.0, 40.5[[40.5, 121.4[[121.4, 404.7[[404.7, 2023.4[≥ 2023.4
Category	A	B	C	D	E	F	G
Corresponding IEP	None	low (1)	minor (2)	moderate (3)	severe (4)	very severe (5)	very severe (5)

Table 3: Values of fire sizes used to determine a class of fire danger. The fire size classes follow the US classification and a correspondence of fire danger classes was made between DeepFire and IEP in the present study.

308 Categories of DeepFire values were determined based on thresholds of
 309 burned surface area to provide maps with a categorical color scale for easier
 310 interpretation. For simplicity, the thresholds of the US classification of fire
 311 size⁵ were chosen, providing seven categories ranging from A to G, as reported
 312 in Table 3. This choice is not perfectly suited to DeepFire which estimates the
 313 one-hour fire size of a freely spreading wildfire, whereas the US classification
 314 accounts for all observed fires, even if there were firefighting actions or the
 315 fire did not last one hour. The minimum value returned by ForeFire is
 316 0.45 ha, which implies that class A will not be obtained in the present study
 317 at all. Similarly, class G should be quite rare as it is only returned by

⁵<https://www.nwccg.gov/term/glossary/size-class-of-fire>, last accessed on 2020.12.11

318 DeepFire given extreme input conditions (very high wind speed and very low
319 FMC). Still, classes B to F should all contain at least a fair proportion of
320 DeepFire predictions so this choice, although arbitrary, makes sense as a first
321 approach. In practice, the thresholds should be calibrated in order to provide
322 a fire danger indicator that is as relevant as possible. Finally, for qualitative
323 comparison of danger class predicted by DeepFire and IEP, a correspondence
324 reported in Table 3 was made.

325 **2.3 Probabilistic predictions representing input uncer-** 326 **tainty**

327 A major aspect that is not accounted for by the deterministic predictions
328 is the uncertainty associated to the modelling process, that could lead to a
329 difference between the “actual” one-hour fire size that would be observed and
330 its estimate. Although it may not be possible to determine the actual value
331 (firefighting actions usually occur, it can be difficult to measure, etc.), several
332 sources of uncertainty can be identified such as the error of approximation be-
333 tween DeepFire and ForeFire and simplifying model assumptions. Arguably,
334 the main source of uncertainty regarding DeepFire predictions stems from
335 the weather predictions and values of fuel model parameters, that is to say
336 input uncertainty.

337 Previous studies [Allaire et al. \(2020,2\)](#) focused on the quantification of
338 input uncertainty in simulations of wildland fire spread. To quantify the

uncertainty in the inputs of DeepFire, we use a *calibrated* probability distribution associated to the inputs of ForeFire simulations. The calibration method is detailed in [Allaire et al. \(2021b\)](#); its key aspect consists in using an *a priori* distribution where the marginals of each individual input have higher variance than in [Allaire et al. \(2020\)](#) and including the information of observed burned surfaces of seven Corsican fires. The *probability density function* (PDF) g of the calibrated distribution is intended to be higher than the prior PDF f for a given input vector \mathbf{u} when there is good overall agreement between observed burned surface \mathcal{S}_{obs} and corresponding simulated burned surface $\mathcal{S}_{\mathbf{u}}$. More precisely, the PDF g can be expressed as follows:

$$g(\mathbf{u}) = \frac{e^{-\beta E(\mathbf{u})} f(\mathbf{u})}{\int e^{-\beta E(\mathbf{u})} f(\mathbf{u}) d\mathbf{u}}, \quad (2.2)$$

where $\beta > 0$ and E is a positive “energy” function that is equal to 0 when $\mathcal{S}_{\mathbf{u}} = \mathcal{S}_{\text{obs}}$ for each fire and increases with the dissimilarity between simulated and observed burned surfaces. Measurement of shape dissimilarity relies on the Wasserstein distance, which is a metric that appears in the field of optimal transport (see, for instance, [Santambrogio \(2015\)](#) for an extensive review), and $E(\mathbf{u})$ can be understood as an energy that is required to transform $\mathcal{S}_{\mathbf{u}}$ into \mathcal{S}_{obs} . The definition of g is inspired from Bayes’ rule:

$$p(\mathbf{u}|\mathcal{S}_{\text{obs}}) = \frac{\mathcal{L}(\mathcal{S}_{\text{obs}}|\mathbf{u})f(\mathbf{u})}{\int \mathcal{L}(\mathcal{S}_{\text{obs}}|\mathbf{u})f(\mathbf{u})d\mathbf{u}}, \quad (2.3)$$

where $p(\cdot|\mathcal{S}_{\text{obs}})$ is the posterior PDF that would be obtained from $\mathcal{L}(\mathcal{S}_{\text{obs}}|\cdot)$,

the likelihood of the observation, and f , the prior PDF. The exponential in Equation (2.2) can therefore be seen as a pseudo-likelihood function, whose weight increases with the parameter β . When $\beta = 0$, the calibrated PDF g is equal to the prior PDF f .

In the present study, the calibrated input distribution obtained with $\beta = 1/2$ is used. The components of the input vector \mathbf{u} are similar to the scalar inputs of DeepFire listed in Table 1. The perturbations in fuel parameters are exactly the same and can be used directly as input of DeepFire instead of the values 0 and 1 used for the deterministic prediction. Regarding wind speed and FMC, however, the corresponding components of \mathbf{u} are perturbation coefficients so that each value of W_x , W_y , and m_c in the weather forecast is perturbed using the corresponding components of \mathbf{u} . As for the deterministic prediction, a threshold is applied so that the perturbed value of wind speed norm and FMC fall in the range used to train DeepFire. By sampling n independent sets of perturbation coefficients $(\mathbf{u}_1, \dots, \mathbf{u}_n)$ following the calibrated distribution, one may obtain n replicas of the deterministic DeepFire predictions but using perturbed inputs so that each of these n predictions is different. The resulting ensemble of n DeepFire predictions is referred to as a *probabilistic prediction* in the following.

A probabilistic prediction based on a large ensemble can be fairly easily obtained when one considers a single ignition location. However, computing an ensemble of M high-resolution maps requires M times more computational time, making the generation of a representative ensemble too long for

operational context conditions. To circumvent this issue, the spatial resolution of the fire danger maps can be reduced. For each large cell and each forecast time, we define a “representative cell”: the small cell which has the highest DeepFire value according to the deterministic prediction. For each large cell, the computations of the probabilistic prediction is only carried out for the small representative cell. In case the large cell contains a high number of non-burnable cells, it may be considered as non-burnable as well (here, it is considered to be the case when at least 90% of the cells are non-burnable). It must also be noted that these ensembles are distinct from atmospheric forecast ensembles, while the later may also be used as inputs for these simulations to account for weather uncertainties, all fire danger predictions for a given day in the present study, whether deterministic or probabilistic, were based on a single deterministic weather forecast.

2.4 Application to weather forecasts corresponding to 13 Corsican fires

Daily forecasts are generated with a high-resolution run of the limited area model Meso-NH version 5.4 [Lac et al. \(2018\)](#). While being the support of coupled fire/weather simulations, Meso-NH is also the atmospheric research model of the French community, maintained and developed by two laboratories associated on the research program (Centre National de la Recherche Scientifique and Laboratoire d’Aérodologie) that provided access to opera-

402 tional runs providing the boundary and initial conditions. Boundary and
 403 initial conditions are gathered from the Météo-France archive of the AROME
 404 model [Amodei et al. \(2015\)](#). The 00Z run is downloaded daily from the sys-
 405 tem and is available at around 3AM. The daily computation is performed
 406 by Meso-NH on a Cartesian regular grid at 600 meter resolution and 46 at-
 407 mospheric levels (up to an altitude of 7000 meters). In the application over
 408 Corsica this resulted in a horizontal grid of 300 by 180 points. Overall com-
 409 putation of the 42 hours of forecast takes an average of 2 hours on 360 CPU.
 410 In these runs, Meso-NH is run coupled to ForeFire [Filippi et al. \(2010\)](#) code
 411 in order to provide high frequency (2 minutes) model state outputs, resulting
 412 in 252 output files containing water, temperature, cloud fraction, turbulence
 413 and U,V and W wind fields. Such high frequency outputs is an important
 414 point to, at the same time, display the complexity of local situations that
 415 can locally be very perturbed by orography and rapidly changing situations
 416 that may be critical on the field.

417 Still, as these forecasts cover more than one day, we focused more partic-
 418 ularly on the prediction between T+6 h and T+30 h. The spatial resolution
 419 of the probabilistic DeepFire predictions is 640 m to reduce computational
 420 time, instead of 80 m for the deterministic predictions (cf. Section 2.3). This
 421 allows to compute an ensemble of 100 maps in a time that is about twice
 422 as long as for computing a deterministic map (presented computed duration
 423 for non-emulated maps are estimations based on a limited number of sim-
 424 ulations, full computations were not run). For specific ignition locations, a

larger ensemble of size 10,000 is computed as well. The characteristics of both deterministic and probabilistic predictions are summarized in Table 4.

Type of prediction	Ignition points	Spatial resolution	Total number of time steps	Number of time steps in 24 hours	Ensemble members
Deterministic	1.2×10^6	80 m	253	145	1
Probabilistic	2.0×10^4	640 m	253	145	100 (10,000 locally)

Type of prediction	Total number of fire danger estimations	Computational time DeepFire, total	Computational time ForeFire, total	Computational time DeepFire, 1 forecast step	Computational time ForeFire, 1 forecast step
Deterministic	3.0×10^8	2 hours	4 months	30 seconds	half a day
Probabilistic	5.1×10^8	3.4 hours	8 months	1 minute	one day

Table 4: Characteristics of deterministic and probabilistic predictions and estimation of computational time.

The predictions are carried out and analyzed for 13 daily weather forecasts, each corresponding to the occurrence of a wildfire of 100 ha or more in Corsica island. Data on these 13 fires are available in the French database Prométhée (cf. <https://www.promethee.com/>, last accessed 2021.01.21) and some are reported in Table 5. Regarding the “time of fire alert” reported in Prométhée, one may assume that, before this time, the fire has ignited and started spreading and, although its size at the time is unknown, it may be close to the initial fire size of 0.45 ha assumed in DeepFire estimations. The fire size in Table 5 corresponds to the final burned surface, which results from firefighting actions and fire spread for generally more than one hour, so it does not make much sense to compare it with DeepFire predictions. Depending on the fire, the coordinates of the ignition point may be available, and in any case, its estimated location is at least identified by a square of $2 \text{ km} \times 2 \text{ km}$. When only the latter information is available, it will be

assumed that the ignition point is located at the center of the square. In either situation, the ignition point coordinates may not be perfectly accurate and one may assume that there could be an error ranging between 100 m and 1 km. For the vil_2017 fire case, the fire started on 20-10-2017 (the date indicated in Prométhée) and only spread a little then it started spreading again two days later from which it burned most of the final observed burned surface; therefore, corrected estimations of time of fire start and ignition location are reported for this fire instead of the ones available in Prométhée.

Fire id	Town	Date and time of first alert (UTC)	Fire size (ha)	Fire season
bon_05-2017	Bonifacio	30-05-2017 11h43	380.0	summer
bon_07-2017	Bonifacio	17-07-2017 11h50	121.8	summer
olm_2017	Olmata di Tuda	24-07-2017 10h49	2118.0	summer
pal_2017	Palneca	02-08-2017 13h48	185.5	summer
cal_2017	Calenzana	05-08-2017 15h42	124.6	summer
non_2017	Nonza	10-08-2017 22h45	1617.0	summer
man_2017	Manso	11-08-2017 13h10	109.0	summer
sai_2017	Sainte Lucie de Tallano	15-08-2017 09h42	130.0	summer
vil_2017	Ville di Paraso	22-10-2017 09h00	1517.8	summer
ghi_2017	Ghisoni	26-10-2017 13h37	526.0	winter
san_2018	Saint'Andrea di Cotone	01-02-2018 04h13	1317.2	winter
chi_2018	Chiatra	03-01-2018 18h44	565.7	winter
cal_2019	Calenzana	23-02-2019 19h03	1110.2	winter

Table 5: Information on the 13 fires studied, sorted in chronological order. The “Fire id” is used to identify the fires in the present study, the one in bold being the most detailed in the following sections.

The 13 fires studied all occurred between May 2017, date from which the weather forecasts were running, and late 2019. This relatively long data

451 collection period ensures a relative representativeness of high fire danger
452 meteorological conditions. Among these 13 fires, nine of them are considered
453 to have occurred during hot, windy and dry events in the summer fire season,
454 and the remaining four during windy events in winter fire season. In the
455 present study, no weather forecast corresponds to a day without at least one
456 big fire. In these situations, one may consider that fire danger was high at
457 least at some point during the event for a fire size of at least 100 ha to be
458 observed, especially during the summer fire season during for which more
459 firefighting forces are mobilized to carry out the initial attack as early as
460 possible. Availability of firefighting forces are obviously a strong factor in
461 having a reduced burned area, a notable example being bon_05-2017, a fire
462 happening on a low-danger day in May (no fire brigades were deployed in
463 the territory), yet in a rather remote area so that the fire propagated freely
464 for several hours and was barely fought.

465 Among those 13 fires, olm_2017, non_2017, vil_2017, chi_2018, san_2018
466 and cal_2019 are wind-driven fires, but with notable differences concerning
467 the last 3 winter fires that were the result of extremely high ($>150 \text{ km h}^{-1}$)
468 wind creating intense hot and dry down slope conditions. Ghi_2017 is a
469 different situation, with a rather high wind and dry day (not an unusual
470 situation in autumn) but a fire started in large areas of grounded deciduous
471 leaves that were not yet wet as no rain had fallen for a month. This created
472 a not very intense fire, but with a lot of spotting in a mountainous area that
473 were difficult to fight for several days. Other situations were typical of hot

474 dry day with some winds with less danger and resulted in smaller burned
475 area due to the efficiency of fire fighting.

476 This database is yet representative of 3 years of active fire seasons with
477 unusual, mild and severe cases, chosen thereof as a first approach to evaluate
478 the ability to predict high danger and analyze predictions obtained using
479 DeepFire. Although this implies that the database is somewhat biased, both
480 winter and summer seasons are represented and the weather conditions are
481 quite diverse from a case to another. Also, given the time range of the
482 forecasts, there are also moments when the predicted weather conditions are
483 less prone to fire spread.

484 **3 Analysis of predictions for specific fires**

485 Although IEP and DeepFire serve similar purposes, results are not directly
486 comparable due to their different design. This section tries to tackle this
487 issue by analyzing fire danger predictions, illustrated by figures, that address
488 specific questions. Given the ability of the fire danger prediction system
489 to provide very local values, we first try analyze what is the predicted fire
490 danger in the neighborhood of the ignition location (cf. Figure 5). Then,
491 by leveraging the high temporal resolution, we investigate how the predicted
492 fire danger evolves over time, locally (cf. Figure 4 & 5), and for the whole
493 simulation domain Figure 6. Using ensembles it is also possible to estimate
494 how uncertain the prediction is (cf. Figure 8 at the ignition location and

time of fire alert, Figure 7 at the same location over time, Figure 9. and on the whole domain, cf. Figure 11, 12, 13 at the time of fire start).

Comparison with the IEP as a reference danger rating system is also of importance but not only as a pure numerical output. As being part of a differently designed system, the IEP does not address the same issues. While investigating the ability to forecast local danger, the following comparative analysis also tries to illustrate the interest of having model outputs that are both probabilistic and quantifiable in hectares.

The first part of this section illustrates this difference in designs with a detailed analysis a single case (Calenzana 2017) before a mode synthetic comparison is presented for the other 12 cases.

3.1 A detailed fire case: Calenzana 2017

Calenzana 2017 fire took place at 15h42 (UTC) on August 5th, and burned a modest 119 ha but the most part was burned in less than two hours. Figure 2 presents the meteorological situation that rapidly changed in the area, thus creating a high weather fire danger. At 12:00 a sea breeze was keeping temperatures and air relative humidity within a typical summer day, nevertheless, south-westerly winds strengthened to more than 10 m s^{-1} at 12:30, generating a down slope Foehn effect that took over the sea breeze with higher temperatures and air moisture under 30% at 13:00, which persisted up to the time of reported ignition at 15:40. The phenomenon was also observed at the nearby weather station of Calvi Airport Figure 3 although a

517 little later and more quickly than in forecast.

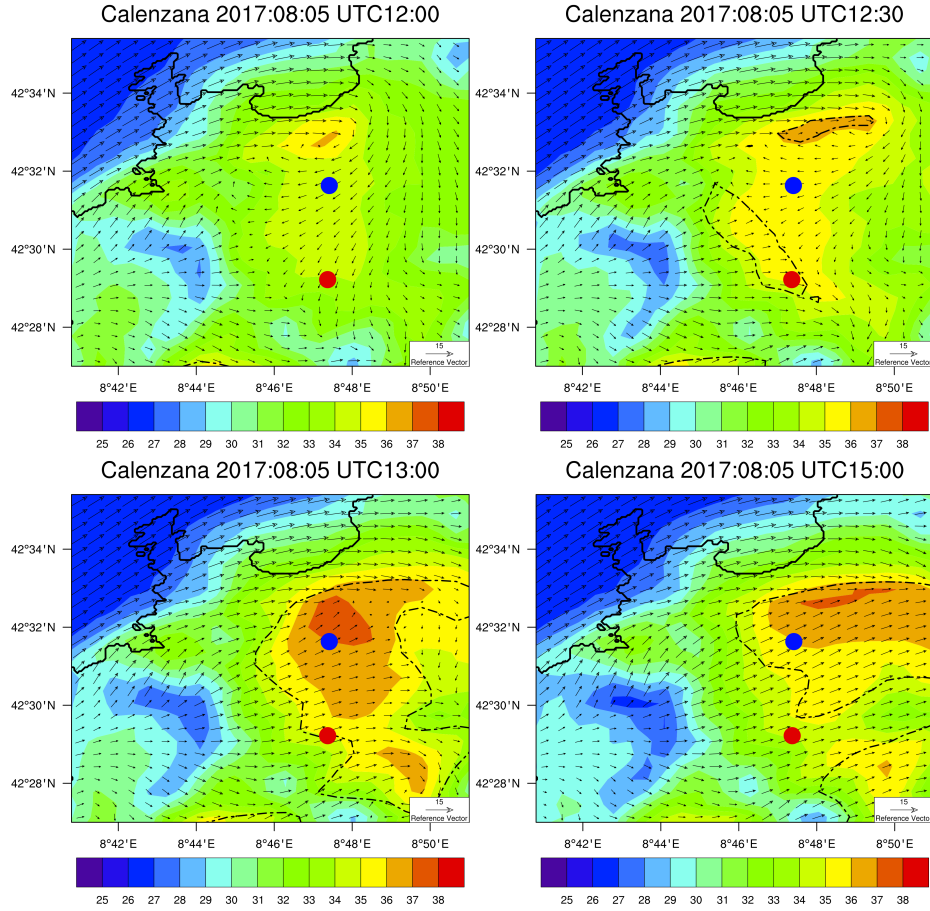


Figure 2: Model forecast at 12:00, 12:30, 13:00 and 15:00 on 08-05-2017 with surface temperature (color), wind (arrows) and area under 30% of relative humidity (dashed lines). Blue dot is the location of Calvi Airport, red dot is the fire ignition location.

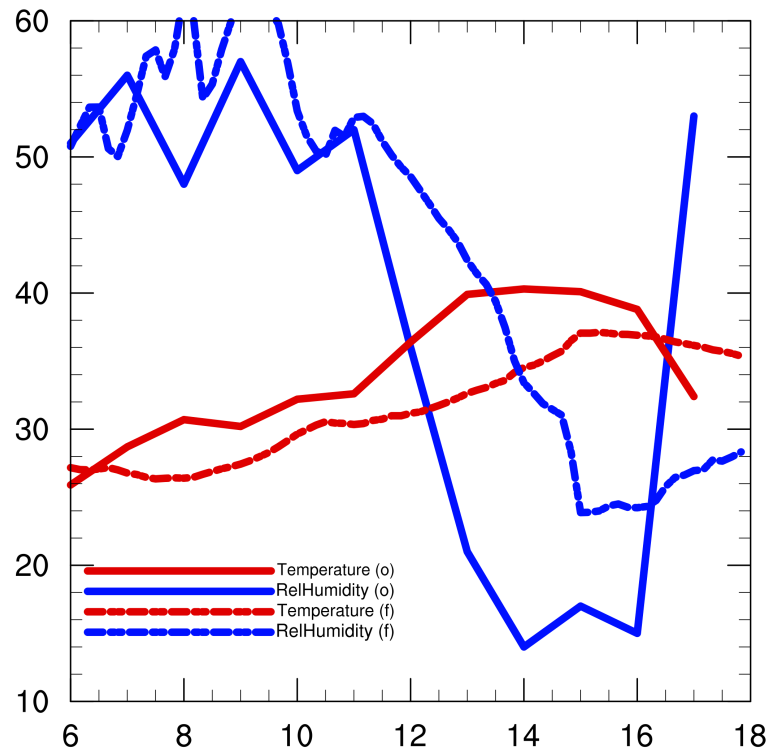


Figure 3: Relative humidity (blue) and temperature (red) observations (plain lines) and forecasts (dashed lines) at Calvi airport (blue dot in Figure 2) on 08-05-2017.

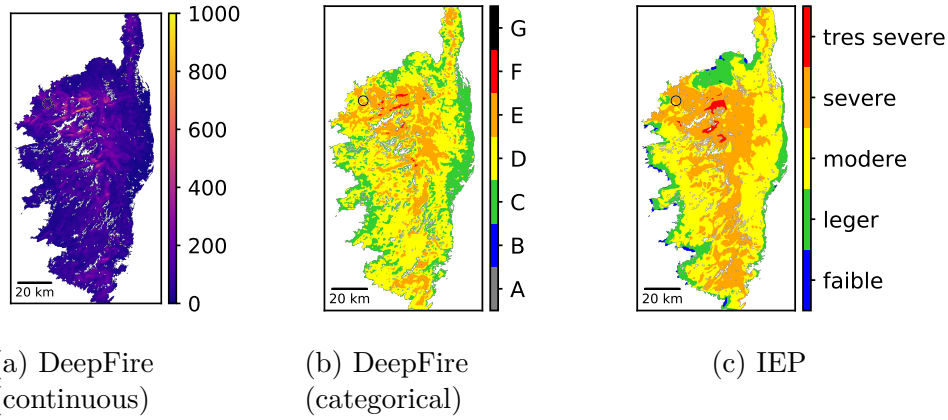


Figure 4: Deterministic maps predicted based on the weather data of forecast time T+15:40 UTC (right before presumed fire start). In white are the locations that are not vegetation, where it is assumed that neither ignition nor spread is possible.

The black circle indicates the presumed location of the ignition point.

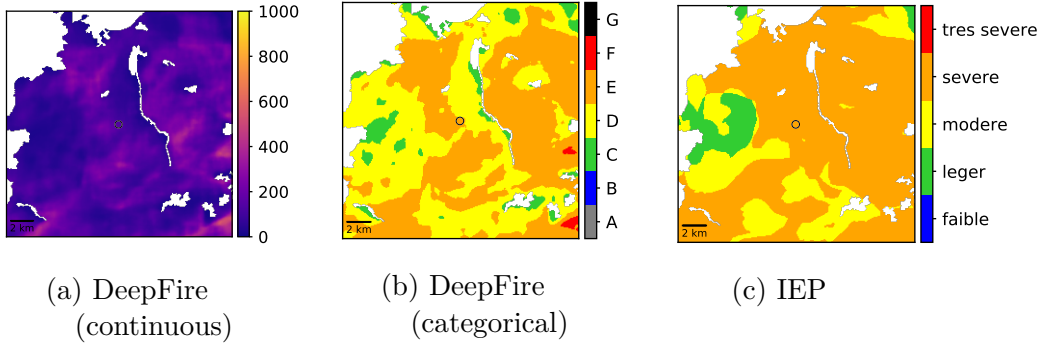


Figure 5: Same as Figure 4 but zoomed around the ignition point

518 The deterministic fire danger maps of both DeepFire and IEP predicted
 519 at the time of fire start, are represented in Figure 4. The DeepFire value in
 520 hectares is shown in Figure 4a with a continuous color scale. The locations
 521 in white indicate the absence of vegetation, where it is assumed that neither

522 ignition nor spread is possible. Among the locations where fuel is present,
 523 the prediction ranges in three orders of magnitude in this specific map (from
 524 less than 1 ha to several hundreds), making it difficult to highlight the differ-
 525 ences in fire danger with a continuous color scale. This appears more clearly
 526 with the categorical color scale in Figure 4b, where most of the island is in
 527 class D or E. Based on the correspondence between danger classes, the same
 528 can be seen on the IEP map shown in Figure 4c. Although these two maps
 529 do not perfectly match, their respective categories were determined indepen-
 530 dently, so we could expect more blatant differences. This overall similarity
 531 is probably due to the fact both indices are very sensitive to wind speed
 532 and FMC. A notable difference is that the map of IEP seems more regular
 533 than that of DeepFire. This is most likely due to the fact that the “driving”
 534 input field of IEP is the Meso-NH forecast whose original spatial resolution
 535 is 600 m, while DeepFire also accounts for a terrain field at an original spa-
 536 tial resolution of 80 m. Another consequence of the sensitivity of DeepFire
 537 to the terrain is more visible in Figure 5 which is zoomed around the ig-
 538 nition point. In Figure 5b, the ignition point is in class E, but slightly to
 539 the north-east of this point, some locations are in class D or even C. This is
 540 due to the forecasted wind in this region, which is oriented to the north-east
 541 where there is a long area from north to south with no vegetation where the
 542 fire cannot spread, hence the lower potential fire size at the aforementioned
 543 locations. In Figure 5c, it clearly appears that the IEP does not account for
 544 this characteristic, as these locations are in the same danger class as that of

545 the ignition point (4: “severe”).

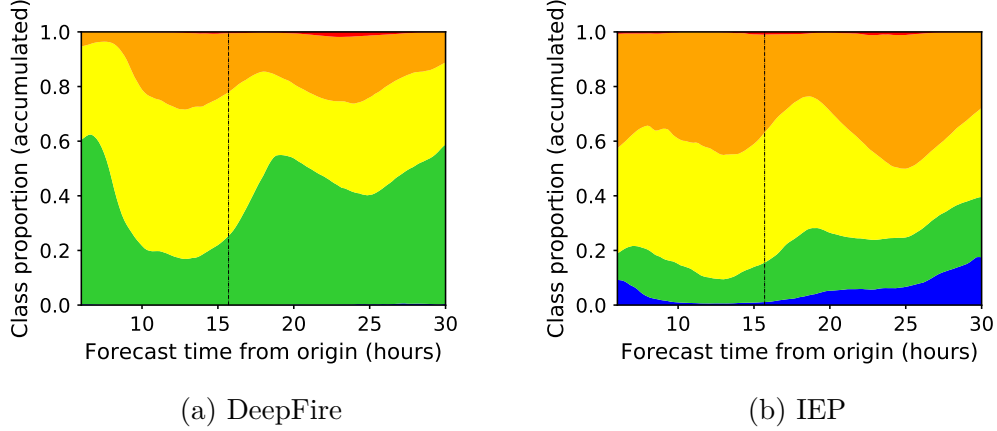


Figure 6: Proportion of points on the island predicted in a given area/danger category as a function of forecast time between T+6 and T+30. The vertical line indicates the time of fire start.

(a) Categories of burned surface area range from A to G, cf. colorbar in Figure 4c.

(b) Categories of IEP range from “faible” to “très sévère”, cf. colorbar in Figure 4b.

546 To summarize the overall of fire danger on the island, one may compute
 547 the proportion of locations in each danger category. This “spatial” distribu-
 548 tion is represented between T+6 and T+30 in Figure 6 for both DeepFire
 549 and IEP categories. The proportion of locations in DeepFire class E is lower
 550 than the proportion for IEP class 4 at all times, but interestingly, they in-
 551 crease or decrease at almost the same times. For both indices, fire danger
 552 seems to decrease on most of the island around the time of fire start, whereas
 553 the evolution of both deterministic and probabilistic DeepFire predictions at
 554 the ignition location represented in Figure 7 shows the opposite trend. At

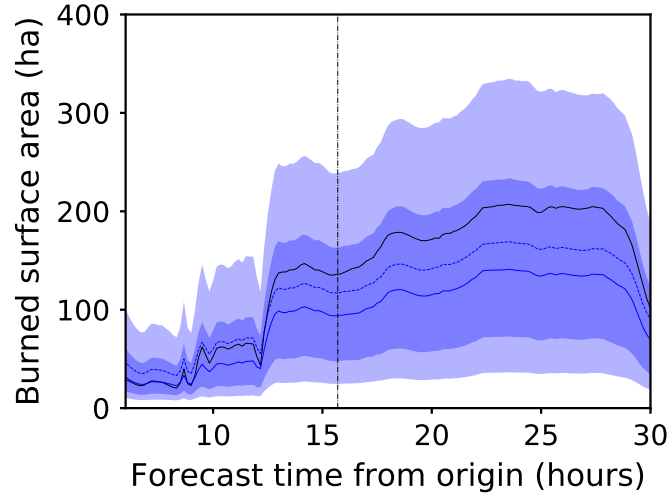


Figure 7: Ensemble of 10,000 emulated burned area (in hectares) predicted at the ignition point as a function of forecast time between T+6 and T+30. The vertical line indicates the time of fire start.

Solid black line: deterministic prediction; solid blue line: mean; dotted blue line: median.

Shaded areas delineate quantiles: first and third quartiles (dark blue); and the first and ninth deciles (light blue).

555 this location, the DeepFire deterministic prediction is fairly low until T+13
 556 where an abrupt increase by about 100 ha occurs in only one hour and stays
 557 in class E until it decreases at the end of the night. This sudden increase
 558 is yet well-connected with the local conditions and the sudden increase in
 559 temperature and lower relative humidity occurring locally near the ignition
 560 area as exposed in Figure 2.

561 Focusing on the distribution of DeepFire values, the histogram of the
 562 probabilistic prediction obtained at the ignition location at the time of fire
 563 start is represented in Figure 8. The raw histogram is right-skewed and has a

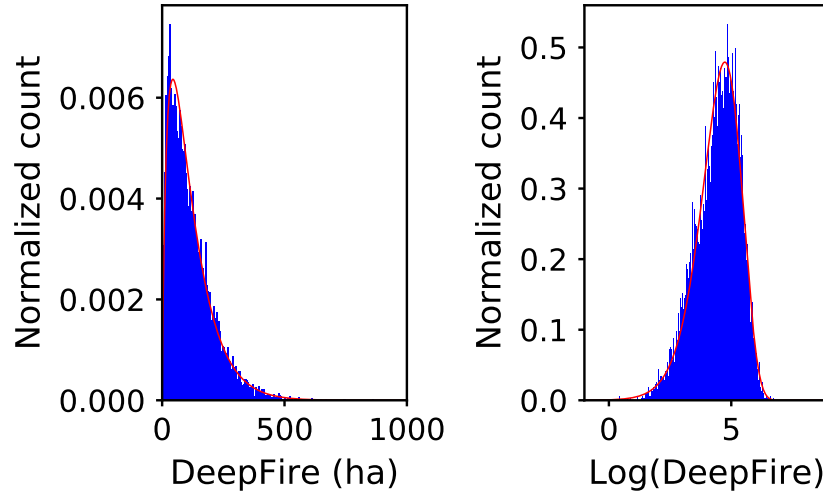


Figure 8: Distribution of the ensemble of 10,000 DeepFire values predicted at the time of fire start at the ignition location.

The red curves are the PDFs obtained from fitting the parameters of a Johnson's S_U distribution to the logarithm of DeepFire values by maximum likelihood estimation.

single mode; it may resemble a log-normal distribution at first glance, but the empirical distribution of its logarithm is fairly left-skewed instead of being symmetrical. At several other time steps and/or locations, however, a log-normal distribution fits the predicted DeepFire ensemble quite well, as shown in Figure 9a. A good agreement was obtained for most DeepFire ensembles when fitting the parameters of a Johnson's S_U distribution to the logarithm of DeepFire values by maximum likelihood estimation. This family includes normal distribution, hence the good fit in several occasions, but is also well-suited to the left-skewed distribution obtained at the ignition location and time of fire start. Still, although a Johnson's S_U distribution generally allows

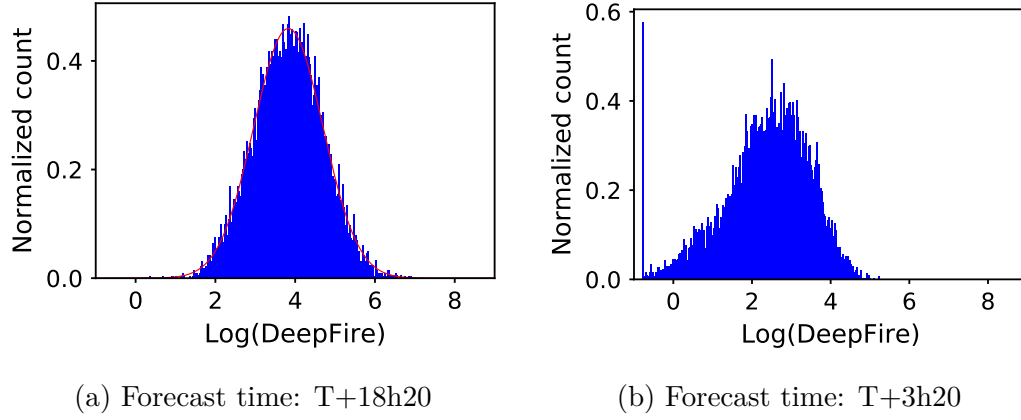


Figure 9: Distribution of the ensemble of 10,000 logarithm of DeepFire values predicted for the cal_2017 fire case. Contrary to Figure 8, the location is not the ignition point.

(a) The red curve is the PDF obtained from fitting the parameters of a normal distribution to the logarithm of DeepFire values by maximum likelihood estimation.

for a decent agreement, the shape of the fitted PDF associated to either the raw DeepFire value or its logarithm could match the histogram better around the PDF's mode when the distribution is skewed as in Figure 8. Also, the ensemble in Figure 9b shows that the minimum DeepFire value (0.45 ha) may be predicted for several ensemble members, resulting in a Dirac at this value that a Johnson's S_U distribution (which is continuous) cannot account for.

The distribution of DeepFire ensembles cannot be simply summarized by a simple distribution such as a log-normal. However, focusing on the ensemble mean and standard deviation reveals that over the time range of the forecast, the coefficient of variation of the DeepFire ensemble predicted at the ignition location remains between 0.7 and 1.1. For other times and/or

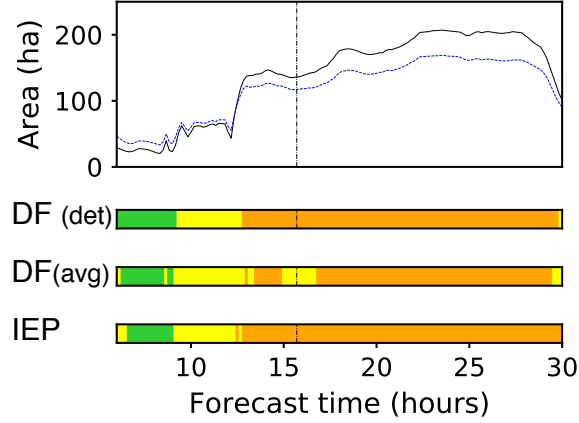


Figure 10: Predictions at the ignition point as a function of forecast time between T+6 and T+30. The vertical line indicates the time of fire start. Top most: DeepFire both deterministic (solid black) and 10,000-size ensemble mean (dotted blue). Horizontal colorbars, from top to bottom: category of the DeepFire deterministic area; category of the DeepFire mean area; IEP

585 locations, the standard deviation of the ensemble is in the same order of
586 magnitude as its mean as well, so one may focus on the latter for simplicity.
587 The evolution of the ensemble mean at the ignition location is represented
588 in Figure 10 with the deterministic prediction, showing their respective fire
589 danger categories, as well as that of the deterministic IEP for comparison.
590 This “concise” plot reveals that the deterministic value and the ensemble
591 mean are predicted in the same category and match with those of IEP for
592 most of the forecast.

593 The probabilistic DeepFire prediction on the whole island consists in a
594 smaller ensemble (100 members) than the local one that was analyzed previ-

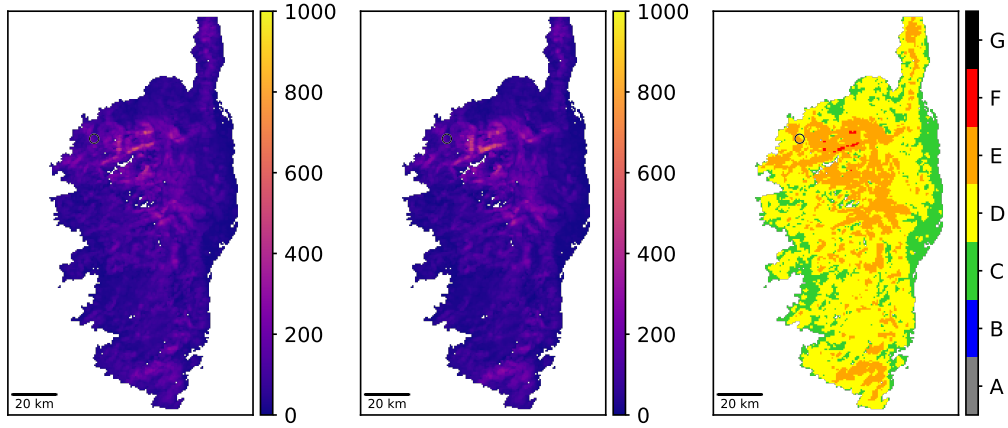


Figure 11: “Probabilistic” predicted maps at forecast time 15h40 UTC (right before fire start). From left to right: mean (continuous), standard deviation, mean (categorical).

ously (10,000 members). The ensemble of DeepFire maps predicted for the time of fire start is summarized in Figure 11 by the ensemble mean and standard deviation computed for each location. Overall, the higher the mean, the higher the standard deviation, which was also the case for the large ensemble at the ignition location. More precisely, the coefficient of variation is close to unity and for almost all locations and forecast times, it is contained in $[1/5, 5]$ in this case. Focusing on the map of ensemble mean represented with a categorical color scale, it appears similar to its deterministic counterpart represented in Figure 4b and does not provide much additional information.

In order to show the uncertainty represented by the probabilistic prediction by means of maps, one may consider other quantities than the ensemble mean and standard deviation. For instance, the fire size category with highest probability (aka, “most likely category”) is computed for each location

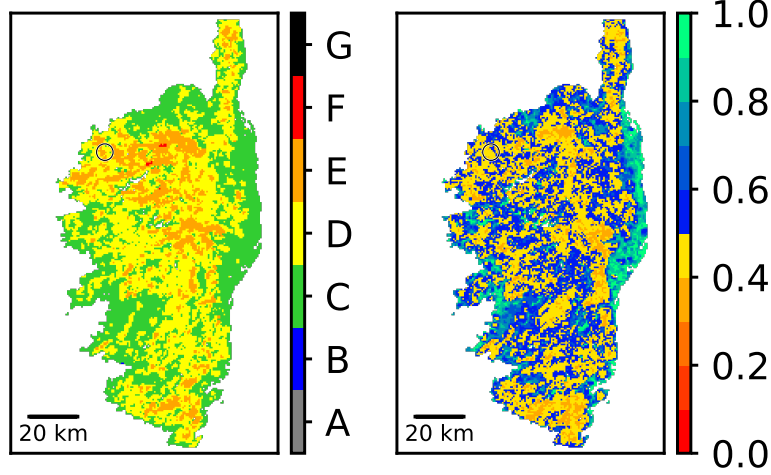


Figure 12: Most likely DeepFire size category in the ensemble (left) and associated probability (right)

and shown in Figure 12 together with the probability of this category. Like the ensemble mean, the map of most likely category is similar to its deterministic counterpart in Figure 4b. Regarding the associated probability, most locations have a probability ranging in $[0.3, 0.7]$. Locations whose most likely category is C tend to have probabilities closer to unity, so despite providing an intuitive representation of uncertainty, these maps do not seem to provide useful information.

Arguably, the probabilistic prediction is best represented by maps of quantiles computed for each location. Using the same quantiles as in Figure 7 as well as the ensemble minimum and maximum, the resulting map are shown in Figure 13. These maps can be understood as predicted scenarios that range from most optimistic (ensemble minimum at the left) to most

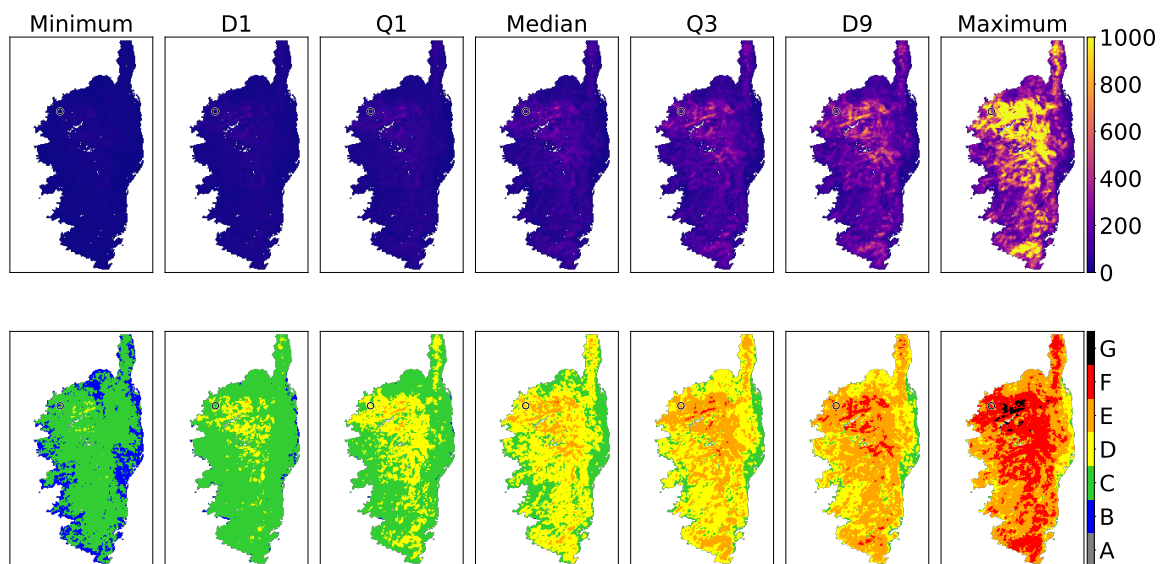


Figure 13: Statistics of the ensemble of DeepFire values predicted for the time of ignition mapped over Corsica island (Calenzana 2017.08.05)

Upper row: continuous; lower row: categorical

Q1 and Q3: first and third quartiles; D1 and D9: first and ninth deciles.

620 pessimistic (ensemble maximum at the right). It is arguably a more intuitive
 621 way to represent the uncertainty in the prediction than with the standard
 622 deviation or the probability of the most likely category. In this case, it seems
 623 that the map of minimum and, to some extent, the map of first decile show
 624 scenarios where fire danger categories are underestimated. Likewise, the map
 625 of maximum overestimates fire danger, but the other maps where fire danger
 626 is not as extreme seem fairly relevant.

3.2 Overview of predictions for all 13 fires

The fire danger predictions for fire case cal_2017 that were presented in detail in Section 3.1 were also carried out for the 12 other fire cases listed in Table 5. The present section summarizes results for all 13 fire cases.

Fire id	IEP	DeepFire (ha)	Mean (ha)	[Q1, Q3] (ha)	$\mathbb{P}[B]$	$\mathbb{P}[C]$	$\mathbb{P}[D]$	$\mathbb{P}[E]$	$\mathbb{P}[F]$	$\mathbb{P}[G]$
bon_05-2017	3	72.6 (D)	67.2 (D)	[26.3, 86.2]	0.008	0.397	0.464	0.127	0.004	0
bon_07-2017	4	159.8 (E)	143.0 (E)	[60.5, 193.3]	0	0.126	0.422	0.415	0.037	0
olm_2017	4	368.3 (E)	272.5 (E)	[103.3, 362.5]	0	0.048	0.260	0.478	0.214	0.001
pal_2017	4	194.8 (E)	137.0 (E)	[54.5, 159.8]	0	0.139	0.489	0.324	0.048	$< 10^{-3}$
cal_2017	4	136.0 (E)	117.1 (D)	[47.8, 163.1]	$< 10^{-3}$	0.207	0.403	0.377	0.012	0
non_2017	3	118.9 (D)	106.7 (D)	[35.3, 130.8]	0.001	0.292	0.433	0.246	0.028	0
man_2017	2	38.7 (C)	27.9 (C)	[3.45, 32.8]	0.268	0.531	0.163	0.037	0.002	0
sai_2017	4	71.7 (D)	66.7 (D)	[25.3, 83.2]	$< 10^{-3}$	0.439	0.431	0.126	0.004	0
vil_2017	4	453.6 (F)	309.6 (E)	[59.6, 370.5]	$< 10^{-3}$	0.167	0.263	0.344	0.213	0.013
ghi_2017	3	55.9 (D)	63.6 (D)	[32.1, 77.6]	0	0.384	0.518	0.097	0.001	0
san_2018	3	172.3 (E)	144.2 (E)	[49.6, 182.1]	0.003	0.198	0.369	0.378	0.052	$< 10^{-3}$
chi_2018	4	483.0 (F)	312.5 (E)	[79.1, 405.6]	0	0.099	0.272	0.378	0.245	0.005
cal_2019	5	431.9 (F)	450.1 (F)	[181.5, 578.9]	0	0.007	0.121	0.467	0.395	0.010

Table 6: Summary of the fire danger predictions on all 13 fire cases at the presumed location of fire start.

Q1 and Q3: first and third quantiles.

$\mathbb{P}[X]$: predicted probability of being into class X of burned surface area; in bold is the highest probability among all 6 classes.

The deterministic predictions of IEP and DeepFire at the ignition location and time of fire start for each fire are listed in Table 6, together with some statistics of the associated ensemble of 10,000 DeepFire predictions. The DeepFire ensemble mean is generally a bit lower than the deterministic value, except for two fires (ghi_2017 and cal_2019). Both ensemble mean and

636 deterministic value are generally slightly lower than the ensemble's third
 637 quantile, except for man_2017 fire where the deterministic value is slightly
 638 higher. In terms of fire size categories, whether we consider the deterministic
 639 DeepFire value or the ensemble mean, the most likely category is either the
 640 same or one level lower. The most likely category is either C, D, or E, and
 641 in almost all cases, probabilities of being in category B and G are very low.
 642 Comparing the deterministic predictions of IEP and DeepFire, there is an
 643 agreement between fire danger categories for 9 out of 13 cases. Due to the
 644 categorical nature of these ratings, difference of one category are expected
 645 and require a more detailed analysis proposed thereafter.

Fire id	B	C	D	E	F	G	1	2	3	4	5	E+	4+
bon_05-2017	4.6%	39.0%	52.0%	4.3%	< 0.01%	0%	32.1%	40.5%	23.1%	4.3%	0%	4.3%	4.3%
bon_07-2017	< 0.01%	10.0%	52.8%	37.0%	0.2%	0%	0.1%	3.3%	33.0%	63.3%	0.2%	37.2%	63.5%
ohm_2017	5.1%	13.3%	39.7%	40.6%	1.3%	0%	9.1%	22.4%	39.8%	28.4%	0.3%	41.9%	28.7%
pal_2017	< 0.001%	13.1%	47.5%	38.9%	0.5%	0%	< 0.1%	2.1%	18.3%	76.1%	3.4%	39.4%	79.5%
cal_2017	< 0.01%	25.2%	52.6%	21.7%	0.5%	0%	1.1%	14.4%	47.5%	36.0%	1.0%	22.5%	37.0%
non_2017	0.8%	29.9%	34.5%	33.2%	1.6%	0%	14.2%	28.8%	33.9%	22.3%	0.8%	34.8%	23.1%
man_2017	7.9%	8.4%	32.1%	46.2%	5.4%	0%	8.8%	14.8%	41.8%	31.6%	3.0%	51.6%	34.6%
sai_2017	< 0.01%	23.6%	67.8%	8.7%	< 0.001%	0%	0%	5.6%	64.5%	29.9%	0%	8.7%	29.9%
vil_2017	28.6%	17.4%	33.1%	18.8%	2.0%	0%	36.4%	31.2%	26.7%	5.6%	< 0.1%	20.8%	5.6%
ghi_2017	< 0.01%	41.8%	52.6%	5.6%	0%	0%	4.9%	32.0%	53.3%	9.8%	0%	5.6%	9.8%
san_2018	38.4%	11.6%	27.7%	22.3%	< 0.1%	0%	43.4%	39.6%	15.6%	1.5%	0%	22.3%	1.5%
chi_2018	35.0%	6.7%	16.4%	32.8%	9.0%	0%	26.4%	36.4%	18.4%	15.0%	3.8%	41.8%	18.8%
cal_2019	1.1%	9.7%	31.2%	43.2%	14.8%	< 0.1%	1.6%	4.9%	19.6%	55.6%	18.2%	58.0%	73.8%

Table 7: Summary of the fire danger predictions on all 13 fire cases over the whole island at the time of fire start: proportion of each class for both DeepFire (B to G) and IEP (1 to 5). In bold is the highest proportion among all classes.

Last two columns: proportion of locations in DeepFire class E or higher (E+) and proportion in IEP class 4 or higher (4+).

646 Focusing on deterministic prediction at the time of fire start for each
647 fire, the maps of both DeepFire and IEP are summarized in Table 7 by the
648 proportions of locations on the island predicted in either category of both
649 fire danger indices. In all fire cases, the proportion of the island that is in
650 class G is null or quasi-null and in most cases, the respective proportions of
651 class F and class 5 are also quite low. Proportion of class B is low in many
652 cases too, but there are 3 of them where it is higher than 25% because a
653 non-negligible part of the island is predicted with a FMC higher than 0.3.
654 This, together with the predicted probabilities listed in Table 6 reveals a
655 limit of using the fire size classes of the US system to quantify fire danger
656 with DeepFire. The thresholds of class B and G are very restrictive (upper
657 bound of 4 ha and lower bound of about 2000 ha, respectively) considering
658 that DeepFire estimates the potential fire size of a fire that spreads freely
659 during one hour. The same holds to some extent for class F (lower bound of
660 about 400 ha), explaining the lower representativeness of the extreme classes
661 in most cases.

662 Among the 13 cases, the fire danger category that has the highest pro-
663 portion is generally class D for DeepFire and 3 for IEP. Comparison with the
664 local deterministic predictions in Table 6 reveals that, in almost all cases, the
665 deterministic prediction is in equal or higher danger class than the category
666 with the highest proportion over the island for both IEP and DeepFire, the
667 only exception being man_2017 fire case. This means that using either in-
668 dex, the ignition point belongs to the areas that are “highlighted” at the time

669 of fire start, which is a desirable property of a fire danger map. It would be
670 even better to observe the same property at the sub-regional scale because
671 the environmental conditions and values at stake can be quite different from
672 one to another, and make more sense from an operational perspective than
673 the average over the island.

674 The previous tables provide data regarding the prediction at the time of
675 fire start, but the evolution of fire danger over time needs to be assessed as
676 well. For simplicity, we decided to present only two of the figures that were
677 shown in the previous section regarding cal_2017 fire case, for the 12 other
678 cases. The zoom around ignition point of the DeepFire deterministic map
679 at the time of fire start with categorical color scale (cf. Figure 5c) provides
680 a prediction of fire danger at the sub-regional scale, while the “concise” plot
681 in Figure 10 shows the evolution at the ignition point of the deterministic
682 DeepFire prediction, the ensemble mean, and the IEP. These two figures
683 are shown for the 12 fire cases in Figure 14. Similarly to the cal_2017 fire
684 case, lower DeepFire values are obtained close to areas with no vegetation
685 that block fire spread in several instances, but this mainly depends on the
686 direction of wind speed.

687 In the case of bon_07-2017 for instance, the wind comes from the east,
688 so when getting closer to areas without vegetation, lower values (classes D
689 and C) are obtained to the west of the ignition location, but higher values
690 (class E) are obtained eastward. Regarding the DeepFire predictions over
691 time at the ignition location, the deterministic value and the ensemble mean

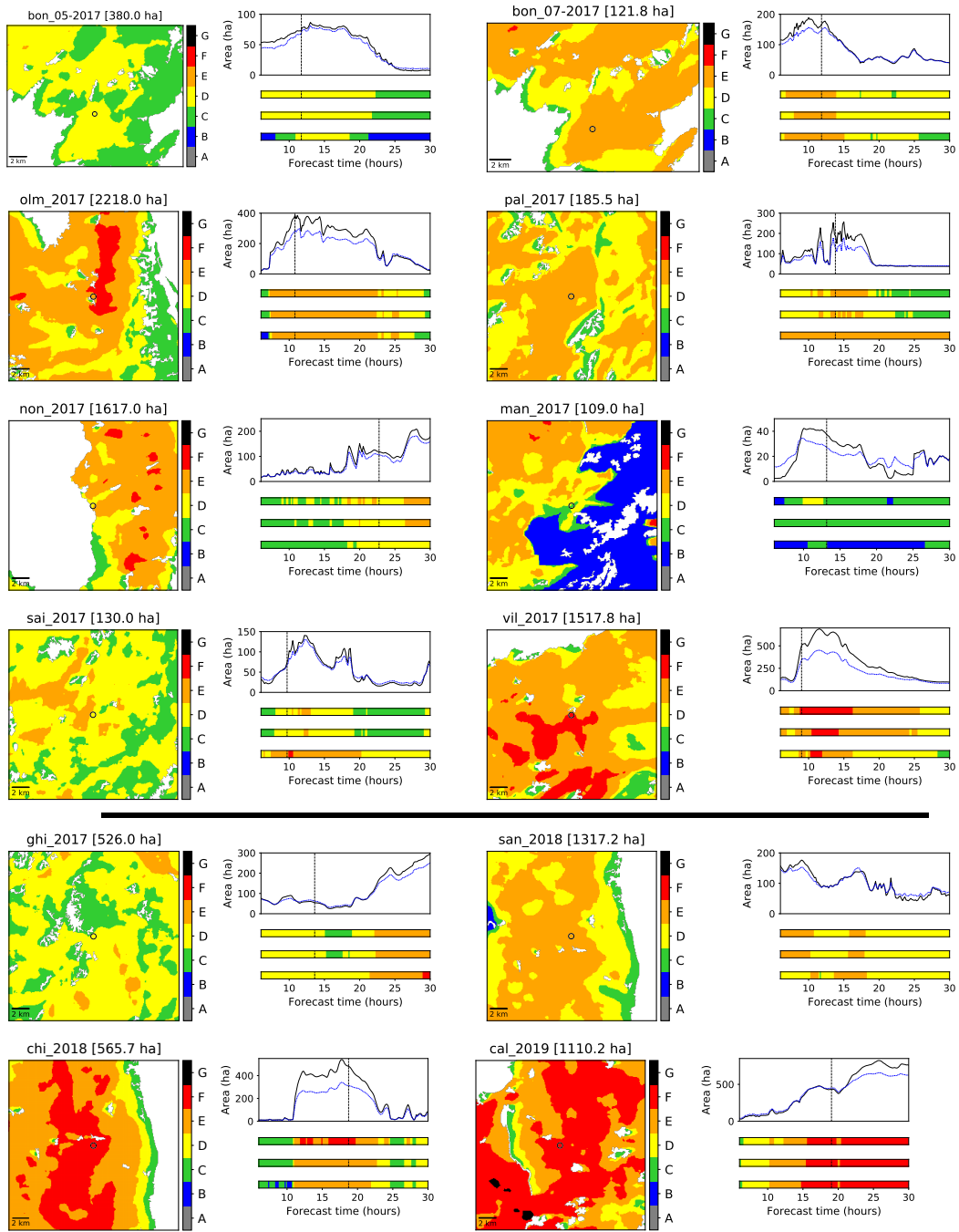


Figure 14: For all 12 complementary fire cases: map of DeepFire predicted at the ignition time zoomed around the ignition location (left, cf. Figure 5c), as well as the “concise” plot for the ignition location (right, cf. Figure 10). Eight top-top-most: summer fires; four bottom-most: winter fires.

are generally close and in the same fire danger category. The ensemble mean seems to follow the same variations with equal or lower magnitude, resulting in predictions in a smaller range than its deterministic counterpart, especially when the latter is very high (> 200 ha). The category of IEP generally matches that of DeepFire, but can be higher or lower by one level for a few hours. There is a notable exception for Sainte Lucie de Tallano where IEP is in the highest class (5) while DeepFire is in class D (2 levels lower), but it can be observed as well that area nearby is only 1 level under (class E).

Looking at the DeepFire deterministic prediction more globally, one may notice that close to the ignition point and/or in a few hours after fire start, higher fire danger classes can be obtained. Five fire cases are concerned: olm_2017 (eastward), non_2017 (eastward and about five hours after fire start), man_2017 (to the north-west), sai_2017 (eastward and after about two hours), and ghi_2017 (about eight hours after fire start). Therefore, considering the vicinity of the ignition point and/or a few hours after fire start, class E or higher is predicted in most cases. This does not happen in two cases: ghi_2017 (class D) where the fire burned slowly and several fire starts occurred, and bon_05-2017 (class D), whose observed fire size (109 ha) is the smallest among the 13 fires studied.

It may make more sense to focus on local predictions in time and space to evaluate predictions of ignition, but DeepFire estimates potential fire spread, so it should be analyzed over a larger range in time and space. For instance, some fires such as non_2017 may spread slowly at first, yet become difficult to

715 stop after some time due to a change in weather conditions and/or when they
716 reach locations with high slope or specific fuel type that are more favorable
717 to fire spread.

718 4 Possible designs for daily fire danger map- 719 ping

720 Focusing on the vicinity of the ignition location and the few hours after
721 fire start, the DeepFire predictions proved to be satisfactory. However, in
722 practice, information on the fires used in the previous section is not known
723 before making fire danger predictions. For the fire cases studied, we knew
724 what were the areas and times to focus on because the information was
725 known *a posteriori*. One could analyze the maps predicted for each time
726 step, but in an operational context one or a few maps that summarize the
727 daily predictions of fire danger. In the present section, we propose several
728 ways of summarizing the predicted fire danger maps and discuss how relevant
729 the resulting daily maps could be in an operational context.

730 The maps proposed in the present section attempt to address several
731 questions. First, what are the locations and time where/when fire danger is
732 highest for the day to come? (cf. deterministic: Figure 15 & 16, and proba-
733 bilistic: Figure 17 & 18) Then, starting from when and for how long is there
734 high fire danger? (cf. Figure 19 and Figure 20.) Regarding these aspects,
735 the question of uncertainty in the prediction is also addressed to some extent

736 by comparing the deterministic maps to their probabilistic counterparts.

737 Similarly to Section 3, several maps are first proposed and detailed for
738 cal_2017 fire case, then a summarized analysis is provided for all 13 fire
739 cases.

740 4.1 Focus on the day of Calenzana 2017 fire

741 From the deterministic prediction of either DeepFire or IEP, the maximum
742 over the day can be computed easily for each location. The maps of the
743 maximum between T+6 and T+30 of DeepFire (resp., IEP) and of the as-
744 sociated time of maximum are shown in Figure 15 (resp., Figure 16). On
745 this fire day, relatively strong south-westerly wind was definitely the driv-
746 ing factor, but this wind also brought some humidity, leaving area of high
747 danger either where a downslope effect was strong. By looking at the time
748 of the maximum of DeepFire on the right of Figure 15 it can be seen that
749 although there is a high danger potential (some areas in red in map on the
750 left), there is actually much contrast regarding time of highest danger in the
751 area of Calvi, suggesting that if there is an event, a detailed local analysis
752 is required. On the IEP map in Figure 16 such requirement is less obvious,
753 with large areas marked in orange and less contrast, indicating a dangerous
754 but more general situation (less discriminant) with an event having strong
755 probabilities of occurring in the morning.

756 If the maximum is obtained at several forecast times, we show the ear-
757 liest one. Because the IEP has only 5 categories, the maximum value will

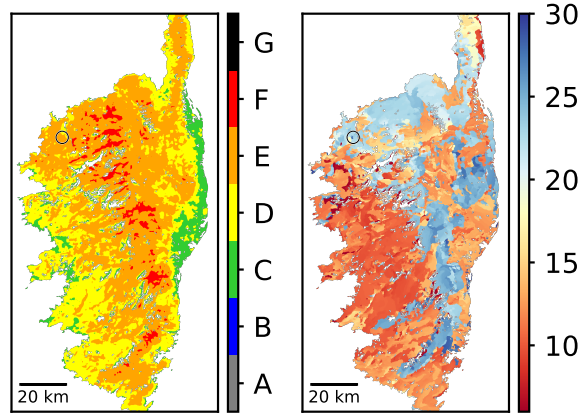


Figure 15: Maximum over the forecast between T+6 and T+30 (left) and time thereof (right) of the DeepFire prediction.

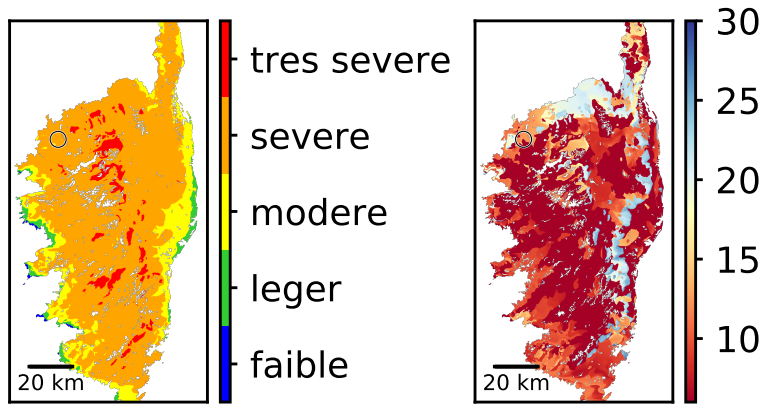


Figure 16: Same as Figure 15 but for IEP prediction

758 most likely be predicted at several times. Figure 16 shows that the time of
 759 maximum is T+6 for most locations, making it hard to tell when fire danger
 760 is highest. In practice, this issue can be avoided by identifying the time of
 761 maximum of another a continuous quantity, such as the FFMC which is one
 762 of the two components of IEP. In Figure 15, although the maximum is rep-

763 resented by a categorical color scale, it corresponds to a continuous index, so
 764 this issue does not occur. The time of maximum for DeepFire predictions is
 765 between T+8 and T+18 on the majority of the island, but for some locations
 766 (even among these predicted in class F) the time of maximum is after T+18,
 767 notably around the ignition location where the fire occurred at T+16 while
 768 the maximum is more around T+22.

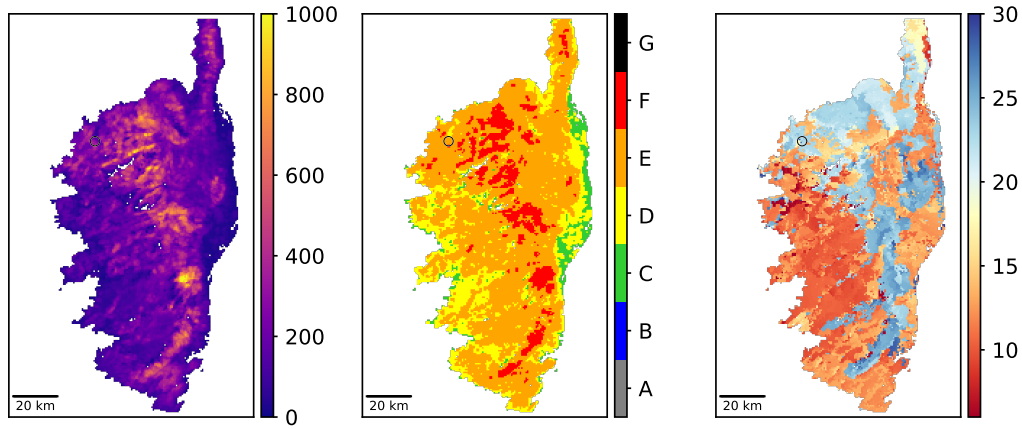


Figure 17: Maximum over the forecast between T+6 and T+30 of the quantile for probability 0.8 in the ensemble of DeepFire predictions.
 From left to right: continuous scale; categorical scale; time of the maximum.

769 From the probabilistic forecast of DeepFire, the computation of a coun-
 770 terpart to Figure 15 is less direct. The ensemble can be summarized by a
 771 statistic such as the mean or a quantile. For either statistic, it makes more
 772 sense to first compute it for all locations and forecast times, then to identify
 773 the time of maximum. In the case of a quantile, for instance, the time of
 774 maximum can therefore be interpreted to that of a more or less optimistic
 775 predicted scenario. In Figure 17, the quantile for probability 0.8 was chosen.

776 According to the Prométhée database, about 80% of the fires in Corsica have
 777 a final burned surface of 1 ha or less, which is quite low considering the range
 778 of DeepFire predictions. Although it is intuitive to define quantile from a
 779 meaningful fire size derived from a database of observations, it does not seem
 780 relevant here, and it makes more sense to interpret the chosen quantile as a
 781 quantity that represents a quite “pessimistic” scenario, yet not too extreme
 782 given the previous analysis of Figure 13. However, as can be seen in Figure 17
 783 (middle map) there are still many locations where fire danger is at least in
 784 class D. It might be possible to make a distinction among the high-danger
 785 areas by looking at the continuous value (left map) less intuitive. Regarding
 786 the time of maximum, it is similar to that of the deterministic counterpart
 787 in Figure 15.

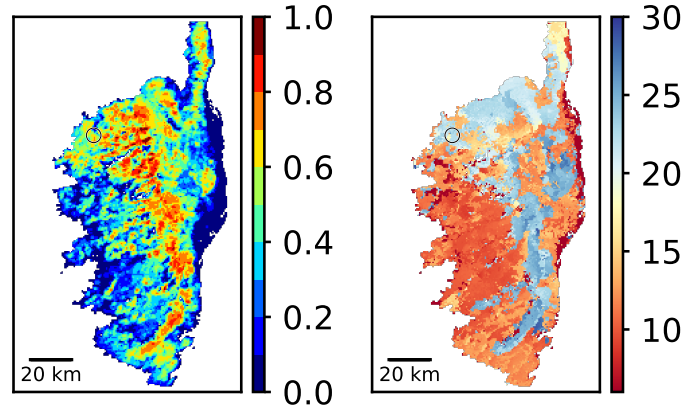


Figure 18: Maximum over the forecast between T+6 and T+30 of the probability of being into class E or higher (left) and time thereof (right).

788 Alternatively, one may consider another maximum over time: that of the

789 probability (based on the ensemble) of being into class E or higher. The
790 resulting map for cal_2017 fire case is shown in Figure 18 together with
791 the associated time of maximum. Compared to Figure 17, the maximum
792 probability seems better suited to discriminate among locations with high
793 fire danger during the day, whereas the time of maximum appears similar
794 overall, except for some locations where the predicted probability is 0 over
795 all 24 hours, resulting in a time of maximum at the value of T+6 by default.

796 Among all the maps presented in the present section up to this point,
797 the maps of time of maximum to point out that the evolution of fire danger
798 over time does not necessarily result in a peak at a single given time (e.g.,
799 midday) everywhere. Still, the time of maximum fire danger might not be
800 the most relevant information for daily predictions as there can be high fire
801 danger for an extended period of time.

802 Regarding the evolution of DeepFire predictions during the day, a more
803 relevant quantity could be the earliest time of being into class E or higher
804 if it happens at all. The associated maps resulting from the deterministic
805 and probabilistic predictions are represented in Figure 19. In the case of the
806 probabilistic version, the earliest time when class E is predicted may differ
807 from a member of the ensemble to another and the class may not be reached
808 for some of them, so the conditional mean among the sub-ensemble of mem-
809 bers for which the class is reached is provided. According to the deterministic
810 version in Figure 19a, there is about a quarter of the island where class E
811 is never reached. However, according to its probabilistic counterpart in Fig-

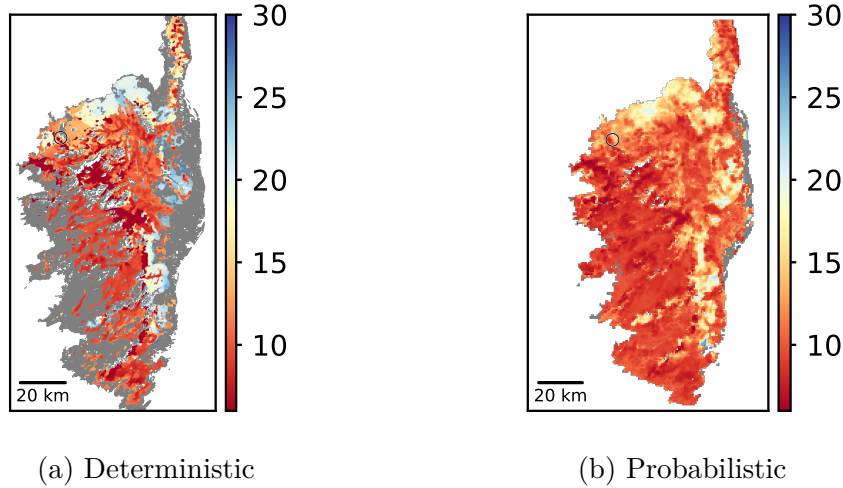
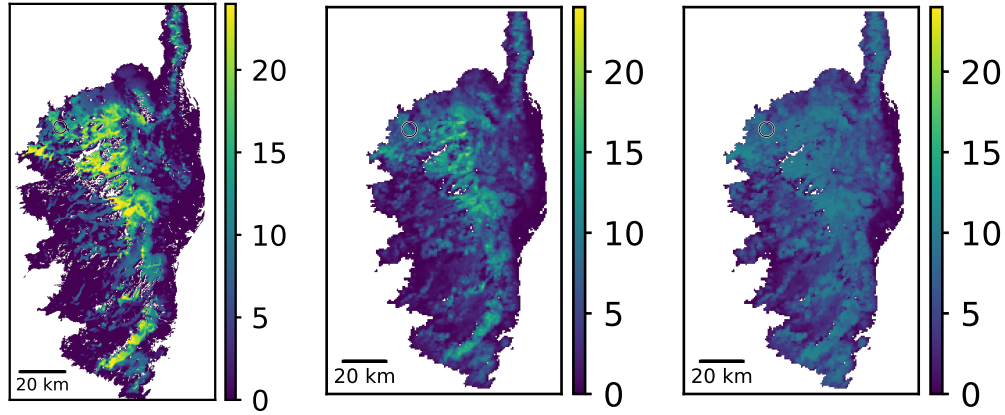


Figure 19: Earliest time between $T+6$ and $T+30$ of a location being predicted into class E or higher (in gray if it does not happen).

The probabilistic version is the conditional mean among the ensemble of the time when class E is reached on the condition that it is reached (a gray area is shown otherwise)

ure 19b, almost all locations have at least one member in the ensemble for which class E is predicted at some point (but the amount of these members may differ between locations), and the map is more homogeneous with fewer extreme values.

To complete the information provided by the previous maps, one may compute the total duration between $T+6$ and $T+30$ when a given location has a DeepFire prediction of class E or higher. In the case of the probabilistic prediction, there is no need to consider a conditional mean as the duration is null if class E is never reached. The deterministic version is shown in Figure 20a, while both mean and standard deviation of the ensemble are shown in Figure 20b to represent the probabilistic forecast. Once again, the map of



(a) Deterministic

(b) Probabilistic prediction

Figure 20: Total duration a location is predicted in class E or higher over the forecast between T+6 and T+30. The colorbar indicates the duration in hours.

(b) Mean (left) and standard deviation (right).

ensemble mean is more homogeneous than its deterministic counterpart and the standard deviation seems higher in locations where the mean is higher.

The analysis of cal_2017 fire case reveals that predicted time of maximum fire danger is similar even when the input uncertainty is accounted for using the calibrated distribution. However, based on the calibrated distribution, the predicted duration of high fire danger is quite uncertain. Such quantity may not serve as a daily fire danger index, but it appears interesting as a complement, because the longer high fire danger conditions last, the more severe the situation may be considered, even if the maximum fire danger category over the day is the same. Still, in the case of DeepFire predictions, not only is the prediction uncertain, but it is also quite sensitive to the choice

834 of the fire size threshold used to define high fire danger categories.

835 4.2 Overview of the predictions for the 12 other fires

836 A number of numerical quantities that could serve as a daily indicator were
837 proposed. Due to its ability to discriminate between locations with high fire
838 danger in the cal_2017 fire case, the maximum over the forecast between
839 T+6 and T+30 of the probability of being into class E or higher is chosen as
840 a daily fire danger map for the 12 other fire cases. This map is shown for all
841 12 cases in Figure 21, together with the evolution of the proportion in the
842 island of each class for both DeepFire and IEP according to the deterministic
843 prediction (cf. Figure 6) to have an idea of how the overall spatial distribution
844 of fire danger evolves over time.

845 Overall there is a general agreement in variations between T+6 and T+30
846 between IEP and DeepFire, with a general trend of more “contrasted” Deep-
847 Fire predictions, indicating a better ability to pinpoint high danger locations
848 (i.e. it is more discriminant).

849 As in Section 3.2, one could present some scalar results regarding the
850 overall spatial distribution of fire danger over time, but considering an av-
851 erage in time and space does not reflect the variations of fire danger and,
852 therefore, does not seem very relevant. Arguably, a daily rating of fire dan-
853 ger based on DeepFire should be computed on a relatively large area, at the
854 sub-regional level for Corsica island and the maximum value over the day can
855 be considered. At this spatial scale, a representative fire danger rating could

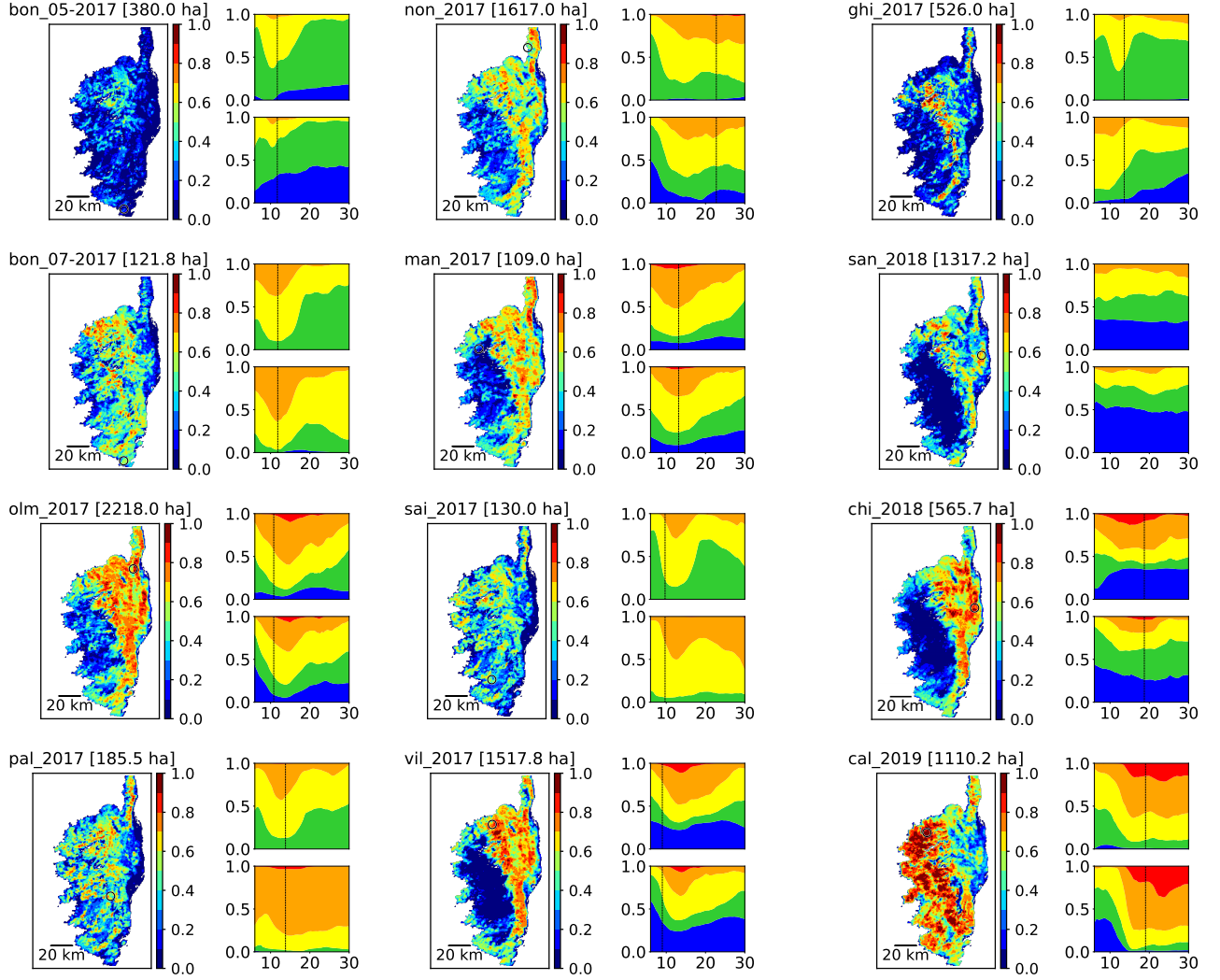


Figure 21: For all 12 complementary fire cases: maximum between T+6 and T+30 of the probability of being into class E or higher (left, cf. Figure 18), together with the evolution of the proportions over the island of DeepFire (top right, cf. Figure 6a) and IEP (bottom right, cf. Figure 6b) classes. First two columns: summer fires; last column: winter fires.

856 be the one associated to the DeepFire value that separates the 80% lower
857 values from the 20% highest in a given area. As a complement to fire danger
858 ratings, that are associated to a low spatial resolution, it makes sense to use
859 the predictions with high spatial resolution and high frequency to analyze
860 the situation in more detail.

861 5 Conclusions and perspectives

862 We have shown a new method to compute fire danger predictions by esti-
863 mating a potential fire size using the deep neural network DeepFire in a
864 reasonable computational time. This allows for the generation of a sequence
865 of high resolution maps of fire danger with high frequency, and an ensemble of
866 such predictions that account for input uncertainty, although at lower spatial
867 resolution. Several quantities derived from predictions of potential fire size
868 can be computed such as maximum fire danger over time, duration of high
869 fire danger, and probabilistic versions thereof using the ensemble. Analysis
870 of the predictions on 13 fire cases showed that, overall, DeepFire predicts
871 higher fire danger in the neighborhood of the ignition location and around
872 the time of fire start. The temporal variation over the day make it difficult
873 to summarize the predictions by a few daily fire danger indices. A few quan-
874 tities were proposed to serve the purpose of providing a daily indicator, but
875 the evolution in time, the granularity, and the uncertainty of the predictions
876 can provide useful information to a forecaster. It is also important to note

877 that all these computations were actually run in a batch processing mode to
878 ensures its potential to run in an operational context.

879 A major strength of the prediction using DeepFire seems to be its spatial
880 granularity allowing to be more discriminant. Compared to traditional fire
881 danger indices that mostly rely on weather forecasts, the potential fire size
882 estimated by DeepFire accounts for the influence of terrain on fire spread at
883 via the variability over space in type of vegetation, presence of non-burnable
884 areas, and slope. The high-resolution maps could be used as complement of
885 fire danger ratings, that generally attribute a single value to a large area, for
886 better anticipation but potentially to help to decide firefighting actions after
887 a fire has started spreading. For instance, the maps can be used to finely
888 identify locations that, if reached at a some point, the fire will spread even
889 faster and become harder to control. Moreover, another strength regarding
890 its design, compared to other fire danger rating systems, is that it is not
891 based on empirical knowledge, except for the actual choices of fire size for
892 each class, these results are not based on experience of past fires, nor on
893 expert analysis.

894 The present study allowed identifying several aspects that could be in-
895 vestigated further or improved. Regarding the DeepFire ensemble at a given
896 time and location, it seems that the logarithm follows a Johnson's S_U distri-
897 bution in several instances, although a more complex family of distribution
898 could lead to a better fit in more instances. Statistical inference of the ensem-
899 bles could also be analyzed at a larger scale, by considering several forecast

900 times and locations, and even the predictions for several days. The analysis
901 of the distribution of numerous potential fire sized predicted by DeepFire
902 could lead to the definition of meaningful thresholds to define categories of
903 fire danger that are better suited than the US classifications which pertains
904 to all observed fires. Defining the thresholds based on observed fire sizes
905 appears difficult in Corsica where a vast majority of fires are attacked early
906 enough to spread far, even when fire danger is high.

907 Information on the intermediate sizes of the fires, rather than that of
908 the final burned surface, would be more relevant for comparison with Deep-
909 Fire, but is difficult to measure. If DeepFire predictions were to be used in
910 operational conditions, the feedback would be very valuable to adjust the
911 thresholds, but also to evaluate the usefulness of such a prediction system.
912 Regarding uncertainty, only a deterministic weather forecasts were used, but
913 probabilistic weather forecasts could be used as an alternative or as a com-
914 plement to represent the probabilistic distribution of wind and FMC inputs
915 of DeepFire.

916 DeepFire focuses on wildfire spread, and could be associated to predic-
917 tions representing ignition probability for a better estimation of fire danger.
918 A model of ignition probability should account at least for weather predic-
919 tions, and other data sources such as proximity to roads or cities could be
920 included to design such a model, which would result in a distribution that is
921 not uniform among burnable locations. Ignition probability is also a major
922 component of wildfire risk quantification frameworks, where DeepFire could

923 be used to estimate fire spread. Although the computational time constraints
924 of fire risk assessment are not as tight as for those of daily fire danger predic-
925 tions, DeepFire could estimate a very large number of potential fire sizes in
926 a reasonable amount of time, much larger than it would be using traditional
927 fire spread simulators, allowing for better and/or more detailed risk quantifi-
928 cation. For a complete assessment of risk, however, the values at stake are
929 not just hectares of land burned and should be accounted for more explicitly
930 than with DeepFire. A step in this direction could therefore be to train a
931 DNN in a similar fashion as DeepFire but based on an estimated cost.

932 Whether it be for fire danger or risk, DeepFire predictions should be also
933 considered and evaluated over a long time period (e.g. a whole year) that
934 includes both days with and without fires occurrence. Due to their ability to
935 account for several types of data, and to carry out fast computations, deep
936 neural networks appear promising to address the complexity of wildfires and
937 design better prediction systems.

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941 Conflicts of interes

942 The authors declare no conflicts of interest.

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